

# Chapter 8 Mixed Geometry and Measurement Problem Solving

## Teacher Guide and Diagnostic Answer Key

### Teacher Guide

#### Teacher Chapter-at-a-Glance

|                               |
|-------------------------------|
| No special materials required |
| Pacing                        |
| Talk Prompt                   |

### Mastery Decision Guide

A missed Tripwire does not automatically mean “redo the chapter.” Confirm the likely misconception, reteach the smallest prerequisite or representation that addresses it, and then use a fresh item before moving on.

Missed Tripwire #1 → The correct number from one measurement is being transferred to a different measurement. Next move: Name the requested measurement first; use “wrap = outside = surface area” before calculating.

### Diagram Production Notes — Chapter 8

Use the diagram master IDs:

Ch19-D01 — Measurement Decision Map

Ch19-D02 — Formula Bank by Measurement

Ch19-D03 — Unit Check Chart

Ch19-D04 — Same Rectangle, Two Questions

Ch19-D05 — Same Cylinder, Two Questions

Ch19-D06 — Composite Shape Strategy

Ch19-D07 — Pythagorean Real-World Triangle

Ch19-D08 — Mixed Trap Visual

The decision map should be clean and teacher-usable. Use short prompts, not long text boxes. The formula bank should be grouped by measurement type, not by chapter. The same-shape diagrams must clearly show that the object is the same while the question changes. The mixed trap visual should help students stop and choose the measurement before calculating.

### Confirmation and Fresh Recheck

Likely evidence: 420 cm<sup>2</sup>.

Possible interpretation: The correct number from one measurement is being transferred to a different measurement.

Confirm with: Ask the student to underline the context word that identifies the measurement and name the measurement before seeing a formula bank.

Targeted repair: Name the requested measurement first; use “wrap = outside = surface area” before calculating.

Fresh recheck: Mix a fencing, painting, filling, and diagonal problem with no topic labels.

## Glossary

apex — top point of a pyramid or cone.

base area (B) — area of the base used in  $V = Bh$  and related volume formulas.

capacity — amount a container can hold, commonly measured in mL or L.

hypotenuse — side opposite the right angle; longest side of a right triangle.

net — flat pattern that folds to form a 3-D solid.

slant height — distance measured along the outside surface of a cone or pyramid; not the vertical height used for volume.

surface area — total area of the outside surfaces of a solid; square units.

vertical height — perpendicular distance from base to opposite face or apex.

volume — amount of space inside a solid; cubic units.

## Index

Capacity — Chapter 7

Composite solids — Chapters 5, 8

Cones, pyramids, spheres — Chapter 7

Nets and 3-D structure — Chapter 3

Pythagorean theorem — Chapters 1–2

Surface area — Chapters 4–5

Volume — Chapters 6–7

Mixed problem solving — Chapter 8

## Common Trap Index

Trap 1 — Using the Pythagorean theorem without a right triangle. Confirm the right angle first.

Trap 2 — Misidentifying the hypotenuse. The hypotenuse is opposite the right angle and is the longest side.

Trap 3 — Calling a cylinder a prism. Prisms have polygon bases and flat side faces.

Trap 4 — Confusing surface area with volume. Name whether the problem asks for outside covering or inside space.

Trap 5 — Counting hidden contact faces in a composite solid. Remove both touching faces from the exposed surface area.

Trap 6 — Using diameter as radius in cylinder formulas. Find  $r = d \div 2$  first.

Trap 7 — Using slant height for volume. Volume uses vertical height.

Trap 8 — Forgetting the one-third factor for pyramids and cones. Compare with the matching prism or cylinder.

Trap 9 — Using square units for volume. Volume uses cubic units.

## Ready to Learn Check

Quick check before starting. A wrong response below points to a prerequisite worth confirming before the chapter builds on it.

1. A fence goes around a garden. Is that perimeter or area?

Correct answer: Perimeter.

Likely wrong answer: Area.

Possible misconception: The context word “around” is not being translated into a measurement.

Next move: Underline fence/around, then name perimeter before calculating.

2. Paint covers the outside of a box. Is that surface area or volume?

Correct answer: Surface area.

Likely wrong answer: Volume.

Possible misconception: Outside covering and inside space are being confused.

Next move: Ask whether paint covers or fills the box.

3. Water fills a tank. Is that surface area or volume?

Correct answer: Volume.

Likely wrong answer: Surface area.

Possible misconception: The context word “fills” is not being translated into volume.

Next move: Use a container and ask what quantity describes how much it holds.

4. A circle has diameter 18 cm. What is the radius?

Correct answer: 9 cm

Likely wrong answer: 18 cm

Possible misconception: Diameter is being used as radius.

Next move: Write  $d = 18$ , then  $r = d \div 2$  before selecting a formula.

Ready-to-Learn item 1

Correct answer: Perimeter.

Likely wrong answer: Area.

Misconception: The context word “around” is not being translated into a measurement.

Next move: Underline fence/around, then name perimeter before calculating.

Ready-to-Learn item 2

Correct answer: Surface area.

Likely wrong answer: Volume.

Misconception: Outside covering and inside space are being confused.

Next move: Ask whether paint covers or fills the box.

Ready-to-Learn item 3

Correct answer: Volume.

Likely wrong answer: Surface area.

Misconception: The context word “fills” is not being translated into volume.

Next move: Use a container and ask what quantity describes how much it holds.

Ready-to-Learn item 4

Correct answer: 9 cm

Likely wrong answer: 18 cm

Misconception: Diameter is being used as radius.

Next move: Write  $d = 18$ , then  $r = d \div 2$  before selecting a formula.

## Practice

*Assign the full set or select items for targeted review. Teacher solutions follow each question.*

**1. A fence around a rectangular yard**

**Solution:**  $1.13 \text{ m}^2$

**2. Carpet for a rectangular room**

**Solution:** 2. area

**3. Paint for the outside of a rectangular prism**

**Solution:**  $3.77 \text{ m}$  Tabletop area:  $r = 1.2 \div 2 = 0.6$   $A = 3.14 \times 0.6^2$   $A = 3.14 \times 0.36$   $A = 1.1304$  The tabletop area is about:

**4. Water to fill a cylindrical tank**

**Solution:** 4. volume

**5. Ribbon around the rim of a circular table**

**Solution:** 5. circumference

**6. Tile for a circular floor**

**Solution:** 6. area

**7. Length of a ladder leaning against a wall**

**Solution:** 7. Pythagorean theorem

**8. Wrapping paper for a gift box**

**Solution:** 8. surface area

**9. Find the perimeter of a rectangle 14 m by 9 m.**

**Solution:** 9.  $P = 2(14) + 2(9)$   $P = 28 + 18$   $P = 46$  46 m

**10. Find the area of a rectangle 14 m by 9 m.**

**Solution:** 10.  $A = 14 \times 9 = 126$  126  $\text{m}^2$

**11. Find the circumference of a circle with diameter 18 cm.**

**Solution:** 11.  $C = \pi d$   $C = 3.14 \times 18$   $C = 56.52$

**12. Find the area of a circle with radius 7 cm.**

**Solution:** 12.  $A = \pi r^2$   $A = 3.14 \times 7^2$   $A = 3.14 \times 49$   $A = 153.86$

**13. A triangle has angles  $48^\circ$ ,  $67^\circ$ , and  $x$ . Find  $x$ .**

**Solution:** 13.  $48^\circ + 67^\circ = 115^\circ$   $180^\circ - 115^\circ = 65^\circ$   $x = 65^\circ$

**14. A quadrilateral has angles  $90^\circ$ ,  $85^\circ$ ,  $120^\circ$ , and  $x$ . Find  $x$ .**

**Solution:** 14.  $90^\circ + 85^\circ + 120^\circ = 295^\circ$   $360^\circ - 295^\circ = 65^\circ$   $x = 65^\circ$

**15. Find the volume of a rectangular prism 10 cm by 6 cm by 4 cm.**

**Solution:** 15.  $V = 10 \times 6 \times 4$   $V = 240$   $240 \text{ cm}^3$

**16. Find the surface area of the same rectangular prism.**

**Solution:** 16.  $SA = 2(10 \times 6) + 2(10 \times 4) + 2(6 \times 4)$   $SA = 120 + 80 + 48$   $SA = 248$   $248 \text{ cm}^2$

**17. Find the volume of a cylinder with radius 4 cm and height 9 cm.**

**Solution:** 17.  $V = \pi r^2 h$   $V = 3.14 \times 4^2 \times 9$   $V = 3.14 \times 16 \times 9$   $V = 452.16$

**18. Find the surface area of the same closed cylinder.**

**Solution:** 18.  $SA = 2\pi r^2 + 2\pi rh$   $SA = 2(3.14)(4^2) + 2(3.14)(4)(9)$   $SA = 100.48 + 226.08$   $SA = 326.56$

**19. A garden is 18 m by 12 m. Find the fencing needed and the area for sod.**

**Solution:** 19. Fencing:  $P = 2(18) + 2(12)$   $P = 60$   $60 \text{ m}$  Sod:  $A = 18 \times 12$   $A = 216$   $216 \text{ m}^2$

**20. A circular table has diameter 1.2 m. Find the edging length and the tabletop area.**

**Solution:** 20. Edging length:  $C = \pi d$   $C = 3.14 \times 1.2$   $C = 3.768$  The edging length is about:

**21. A 13 m ladder leans against a wall. The bottom is 5 m from the wall. How high does it reach?**

**Solution:** 21. The ladder is the hypotenuse.  $5^2 + b^2 = 13^2$   $25 + b^2 = 169$   $b^2 = 144$   $b = 12$  The ladder reaches: 12 m up the wall.

**22. A cylindrical container has radius 40 cm and height 50 cm. Find its volume in  $\text{cm}^3$ , then convert to litres.**

**Solution:** 22.  $V = \pi r^2 h$   $V = 3.14 \times 40^2 \times 50$   $V = 3.14 \times 1600 \times 50$   $V = 251200$  The volume is:  $251200 \text{ cm}^3$   
Since:  $1 \text{ cm}^3 = 1 \text{ mL}$   $251200 \text{ cm}^3 = 251200 \text{ mL}$   $251200 \text{ mL} = 251.2 \text{ L}$

**23. A student uses area to find fencing around a yard. Explain the mistake.**

**Solution:** 23. Fencing goes around the yard, so it needs perimeter, not area.

**24. A student uses volume to find wrapping paper for a box. Explain the mistake.**

**Solution:** 24. Wrapping paper covers the outside of the box, so it needs surface area, not volume.

**25. A student gives a surface area answer in  $\text{cm}^3$ . Explain the unit mistake.**

**Solution:** 25. Surface area uses square units such as  $\text{cm}^2$ . Cubic units are for volume.

**26. A student uses 18 cm as the radius when the diameter is 18 cm. Explain the mistake.**

**Solution:** 26. Diameter is the full distance across the circle. Radius is half the diameter, so the radius is 9 cm.

## Chapter Answer Key

Tripwire Check

The student is not correct.

The volume is:

$$V = 12 \times 7 \times 5 = 420$$

$$420 \text{ cm}^3$$

But wrapping paper needs surface area, not volume.

$$SA = 2(12 \times 7) + 2(12 \times 5) + 2(7 \times 5)$$

$$SA = 168 + 120 + 70$$

$$SA = 358$$

The correct wrapping paper amount is:

$$358 \text{ cm}^2$$

### Worksheet 1: Guided Practice

1.

- a. circumference
- b. area
- c. surface area
- d. volume
- e. perimeter

2.

- a.  
 $P = 36$
- b.  
 $A = 77$

3.

- a. cm
- b.  $\text{cm}^2$
- c.  $\text{cm}^3$

4.

$$r = 8$$

### Worksheet 2: Independent Practice

1.

- a.

$$P = 2(11) + 2(7)$$

$$P = 22 + 14 = 36$$

$$36 \text{ m}$$

b.

$$A = 11 \times 7 = 77$$

$$77 \text{ m}^2$$

**2.**

a.

$$C = 2\pi r$$

$$C = 2(3.14)(5)$$

$$C = 31.4$$

$$31.4 \text{ cm}$$

b.

$$A = \pi r^2$$

$$A = 3.14 \times 5^2$$

$$A = 78.5$$

$$78.5 \text{ cm}^2$$

**3.**

a.

$$35^\circ + 80^\circ = 115^\circ$$

$$180^\circ - 115^\circ = 65^\circ$$

$$x = 65^\circ$$

b.

$$100^\circ + 70^\circ + 110^\circ = 280^\circ$$

$$360^\circ - 280^\circ = 80^\circ$$

$$x = 80^\circ$$

**4.**

a.

$$V = 8 \times 5 \times 3 = 120$$

$$120 \text{ cm}^3$$

b.

$$SA = 2(8 \times 5) + 2(8 \times 3) + 2(5 \times 3)$$

$$SA = 80 + 48 + 30$$

$$SA = 158$$

$$158 \text{ cm}^2$$

5.

a.

$$V = 3.14 \times 3^2 \times 10$$

$$V = 3.14 \times 9 \times 10$$

$$V = 282.6$$

$$282.6 \text{ cm}^3$$

b.

$$SA = 2(3.14)(3^2) + 2(3.14)(3)(10)$$

$$SA = 56.52 + 188.4$$

$$SA = 244.92$$

$$244.92 \text{ cm}^2$$

6.

a.

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

$$c = 10$$

The hypotenuse is:

$$10 \text{ m}$$

b.

$$7^2 + b^2 = 25^2$$

$$49 + b^2 = 625$$

$$b^2 = 576$$

$$b = 24$$

The missing leg is:

$$24 \text{ cm}$$

### Worksheet 3: Mixed Practice & Application

1.

a. Flooring covers the flat floor, so use area.

$$A = 9 \times 7 = 63$$

$$63 \text{ m}^2$$

b. Baseboard goes around the room, so use perimeter.

$$P = 2(9) + 2(7)$$

$$P = 18 + 14 = 32$$

$$32 \text{ m}$$

## 2.

a. Edging goes around the circle, so use circumference.

$$C = \pi d$$

$$C = 3.14 \times 6$$

$$C = 18.84$$

$$18.84 \text{ m}$$

b. Mulch covers the circular region, so use area.

$$r = 6 \div 2 = 3$$

$$A = \pi r^2$$

$$A = 3.14 \times 3^2$$

$$A = 28.26$$

$$28.26 \text{ m}^2$$

## 3.

a. Open-top outside surface area:

Bottom:

$$12 \times 8 = 96$$

Long sides:

$$2(12 \times 5) = 120$$

Short sides:

$$2(8 \times 5) = 80$$

Total:

$$96 + 120 + 80 = 296$$

$$296 \text{ cm}^2$$

b. Volume:

$$V = 12 \times 8 \times 5$$

$$V = 480$$

$$480 \text{ cm}^3$$

## 4.

a. Diameter 12 cm, so:

$$r = 12 \div 2 = 6$$

Open-top surface area:

$$SA = \pi r^2 + 2\pi rh$$

$$SA = 3.14(6^2) + 2(3.14)(6)(20)$$

$$SA = 113.04 + 753.6$$

$$SA = 866.64$$

$$866.64 \text{ cm}^2$$

b. Volume:

$$V = \pi r^2 h$$

$$V = 3.14 \times 6^2 \times 20$$

$$V = 3.14 \times 36 \times 20$$

$$V = 2260.8$$

$$2260.8 \text{ cm}^3$$

c.

$$2260.8 \text{ cm}^3 = 2260.8 \text{ mL}$$

$$2260.8 \text{ mL} = 2.2608 \text{ L}$$

The capacity is:

$$2.2608 \text{ L}$$

or about:

$$2.26 \text{ L}$$

## 5.

The ramp is the hypotenuse.

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

$$c = 10$$

The ramp length is:

$$10 \text{ m}$$

## 6.

The student used diameter as radius.

Radius is half the diameter.

If the diameter is 12 cm, then:

$$r = 12 \div 2 = 6$$

The radius is 6 cm, not 12 cm.

## 7.

Wrapping paper covers the outside of the box, so it needs surface area.

Rice fills the inside of the box, so it needs volume.

The two questions use different measurements because outside covering and inside space are not the same thing.

## Diagnostic Key — High-Leverage Items

### Tripwire Check

Correct answer: No.  $240 \text{ cm}^3$  is the volume. Wrapping needs surface area:  $2(10 \times 6) + 2(10 \times 4) + 2(6 \times 4) = 248 \text{ cm}^2$ .

Likely wrong answer:  $240 \text{ cm}^2$ .

Misconception: The correct number from one measurement is being transferred to a different measurement.

Next move: Name the requested measurement first; use “wrap = outside = surface area” before calculating.

### Tripwire Diagnostic

Correct answer: No.  $420 \text{ cm}^3$  is the volume. Wrapping needs surface area:

$$2(12 \times 7) + 2(12 \times 5) + 2(7 \times 5) = 358 \text{ cm}^2.$$

Likely wrong answer:  $420 \text{ cm}^2$ .

Misconception: The correct number from one measurement is being transferred to a different measurement.

Next move: Name the requested measurement first; use “wrap = outside = surface area” before calculating.

## Exit Ticket Diagnostic — High-Leverage Item

Correct answer: Surface area; square units.

Likely wrong answer: Volume; cubic units.

Misconception: The context word “painted outside” is not being translated into the requested measurement.

Next move: Underline the action word, name the measurement before choosing a formula, then check unit dimension.

## Variation Lab — Expected Ideas

1. Same box, different question: wrapping → surface area (square units); filling → volume (cubic units).
2. Same circle: edging → circumference; covering → area; area requires radius squared.
3. Relevant information is determined by the requested measurement, not by using every number given.

## Transfer Check — Suggested Responses

Familiar: area =  $63 \text{ m}^2$ ; perimeter =  $32 \text{ m}$ . Altered:  $r = 6$ ; outside area =  $\pi r^2 + 2\pi rh = 866.64 \text{ cm}^2$ ; volume =  $2260.8 \text{ cm}^3$ . Non-routine: diagonal =  $34 \text{ cm}$ . Repair: surface area =  $2(60 + 40 + 24) = 248 \text{ cm}^2$ .

## Exit Ticket — Answers

1. perimeter, linear units
2. surface area, square units
3. volume/capacity, cubic units or capacity units as appropriate